

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, JUNE 2022

THIRD YEAR (BATCH 2019-22)

MATHEMATICS (Honours)

Paper : DSE 3 [Topology]

Date : 13/06/2022

Time : 11.00 am – 1.00 pm

Full Marks : 50

Answer all the questions. Maximum you can score is 50.

1. a) Find the derived set of A in (\mathbb{N}, τ) where $\tau = \{ \{n, n+1, n+2, \dots\} : n \in \mathbb{N} \} \cup \{ \emptyset \}$.
b) Show that any topological space is a subspace of a separable space.
c) Is \mathbb{R} with lower limit topology 2^{nd} countable? Justify your answer. [4+5+3]
2. a) Let $g : (\mathbb{R}, \tau_f) \rightarrow (\mathbb{R}, \tau)$ be defined by $g(x) = |x| \forall x \in \mathbb{R}$ where τ_f and τ respectively denote the cofinite topology and the usual topology on \mathbb{R} . Verify whether g is open, closed and continuous.
b) Let $p : \mathbb{R} \rightarrow \mathbb{R}$ be a polynomial. Show that p is a closed map. [(2+2+3)+5]
3. a) Prove that a topological space X is completely regular iff for each subbasic open set S in X and for each $x \in S$, \exists a continuous map $f : X \rightarrow [0, 1]$ such that $f(x) = 0$ and $f(X - S) = \{1\}$.
b) Prove that complete regularity is a hereditary property. [5+3]
4. a) Determine all compact subsets of \mathbb{R} with cocountable topology.
b) Prove that every countably compact topological space is pseudocompact.
c) Give an example of an open cover of $[0, \sqrt{2}] \cap \mathbb{Q}$ having no finite subcover. [4+3+4]
5. a) Show that $A = \{ (x, y) \in \mathbb{R}^2 : x, y \in \mathbb{R} - \mathbb{Q} \}$ is not connected in \mathbb{R}^2 .
b) Show that the components of a topological space make a partition of the space.
c) Find all components of $X = \left\{ 0, 1, \frac{1}{2}, \frac{1}{3}, \dots \right\}$ in the usual topology. [3+5+3]
6. a) Define α^β where α and β are two cardinal numbers.
b) If X has cardinal number α , prove that $\mathcal{P}(X)$, the power set of X has cardinal number 2^α . [2+4]

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